Computational hologram synthesis and representation on spatial light modulators for real-time 3D holographic imaging

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Abstract. In dynamic computer-generated holography that utilizes spatial light modulators, both hologram synthesis and hologram representation are essential in terms of fast computation and high reconstruction quality. For hologram synthesis, i.e. the computation step, Fresnel transform based or point-source based raytracing methods can be applied. In the encoding step, the complex wave-field has to be optimally represented by the SLM with its given modulation capability. For proper hologram reconstruction that implies a simultaneous and independent amplitude and phase modulation of the input wave-field by the SLM.

In this paper, we discuss full complex hologram representation methods on SLMs by considering inherent SLM parameter such as modulation type and bit depth on their reconstruction performance such as diffraction efficiency and SNR. We review the three implementation schemes of Burckhardt amplitude-only representation, phase-only macro-pixel representation, and two-phase interference representation. Besides the optical performance we address their hardware complexity and required computational load. Finally, we experimentally demonstrate holographic reconstructions of different representation schemes as obtained by functional prototypes utilizing SeeReal’s viewing-window holographic display technology. The proposed hardware implementations enable a fast encoding of complex-valued hologram data and thus will pave the way for commercial real-time holographic 3D imaging in the near future.

1. Introduction
Undoubtedly, real-time display holography is one of the most promising developments for the future consumer display and TV market. Over recent years, SeeReal has developed a unique solution to real-time holography, which is ready for the display market [1, 2]. Both real-time computing and particularly a suited spatial light modulator are essential requirements for successful commercialization.

In this paper, we address different issues of hologram encoding methods for representation on customary spatial light modulators (SLM). Although a complex modulator would be most suitable to modify the complex amplitude of a wave field, realistic devices exhibit a certain modulation capability in terms of modulation type (real or amplitude-only, phase-only, coupled), modulation range (restricted phase) and bit depth (number of discrete addressable values). Hologram encoding is therefore directly related to the hardware implementation of the programmable SLM. However, each encoding scheme is subject to approximation errors either caused by the decomposition method itself or by its sensitivity to addressing deviations or quantization errors.
This paper is organized as follows. Section 2 summarizes common definitions, which are applied to evaluate the performance of holographic reconstructions from electrically driven SLMs such as liquid crystal displays. In section 3 we briefly recapitulate SeeReal’s reconstruction scheme using our proprietary Viewing-Window (VW) holography and the methods for hologram calculation by means of Sub-Holograms (SH). Three different encoding schemes are discussed in section 4, including their prototype implementation on SeeReal holographic displays. We end with a comparison of the encoding schemes and concluding remarks.

2. Basic spatial light modulator model
The pixelated structure of an SLM is shown in Fig. 1. It comprises of \( N_x \times N_y \) pixel with an aperture of \( a_x a_y \), which are arranged at a pixel pitch of \( \delta_x, \delta_y \), respectively. Within the aperture area the light can be modulated whereas the region between the pixel is inactive and usually masked for transmissive SLM. The total optical transmittance of a pixelated SLM is characterized by its fill factor \( \mu = a_x a_y / \delta_x \delta_y \). Besides, the fill factor determines the distribution in the far-field diffracted light because the envelope of the reconstructed SLM diffraction orders is defined by the amplitude transmittance of the single pixel. The smaller the fill factor, the more light is distributed into higher orders. The pixelized SLM structure leads to a discrete spatial sampling of an otherwise continuous function, where aliasing has to be prevented. The amount of information that can be recorded in the hologram is directly related to the spatial resolution and the size of the SLM. This fact is represented by the dimensionless space-bandwidth product \( SBP = \nu_x L_x \cdot \nu_y L_y = \frac{f}{2 \delta_x} \cdot \frac{f}{2 \delta_y} = \frac{1}{2} (N_x N_y) \), with \( \nu \) being the maximum spatial frequency according to the sampling theorem and \( L \) the width of the modulator [3, 4].

![Hologram and Field Lens Diagram](image)

**Figure 1.** Geometry of the pixelated SLM and diffraction geometry.

The transmittance function \( t_A(x, y) \) of an ideal pixelated SLM is given by the overall SLM aperture multiplied by the discrete complex amplitude of the pixel matrix [5]

\[
t_A(x, y) = \text{rect} \left( \frac{x}{L_x}, \frac{y}{L_y} \right) \left\{ \text{comb} \left( \frac{x}{\delta_x}, \frac{y}{\delta_y} \right) \cdot h(x, y) \right\} \odot \text{rect} \left( \frac{x}{a_x}, \frac{y}{a_y} \right),
\]

where \( h(x, y) \) is the continuous complex amplitude of the hologram function. Considering SeeReal’s reconstruction scheme using Viewing-Window holography, the diffracted field of the hologram in the Fourier plane is then [5]

\[
U(u, v) = \exp \left[ \frac{i \kappa f}{i \lambda f} (u^2 + v^2) \right] \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} t_0(x, y) t_A(x, y) \exp \left[ -i \frac{2\pi}{\lambda f} (xu + yv) \right] \, dx \, dy.
\]
planar, normally incident and with uniform amplitude. Hence, the complex amplitude distribution in the focal plane is the Fourier transform of the field just behind the SLM multiplied by the quadratic phase term preceding the integral, so we can rewrite $U(x,y) = (i\lambda f)^{-1} \exp \left[ i \frac{\lambda f}{2} (u^2 + v^2) \right] \mathcal{F}\{t_0(x,y) t_A(x,y)\}$.

We now consider the discrete signals existing at the SLM plane and in its Fourier plane, which are $H(x,y) = \text{comb}(x/\delta_x, y/\delta_y) \cdot h(x,y)$ and $S(u,v) = \mathcal{F}\{H(x,y)\}$ for the hologram function and its response in the spectral domain, respectively. As a fidelity criterion of the reconstructed signal in the Fourier domain we apply a scaled signal-to-noise ratio (SNR) defined as [6]

$$\text{SNR} = \frac{\iint |S_n(u,v)|^2 \, du \, dv}{\iint |S_n(u,v) - \beta S_r(u,v)|^2 \, du \, dv}, \quad \text{with} \quad \beta = \frac{\iint \text{Re}\{S(u,v) \cdot S_r^*(u,v)\} \, du \, dv}{\iint |S_r(u,v)|^2 \, du \, dv}, \quad (3)$$

where $S_n$ is the desired, nominal signal and $S_r$ is the real signal at $(u,v)$. The factor $\beta$ represents a constant normalization factor for the power of the real signal $S_r$.

Another fidelity criterion to compare different encoding schemes is the encoding efficiency. It can be either defined in an absolute manner as the fraction of incident optical power on the SLM that appears in the targeting signal window (SW) in the Fourier plane or in a relative way by comparing the optical power in the signal window with those of signal and noise window (NW) together.

$$\eta_{E,\text{abs}} = \frac{\iint_{\text{SW}} |S(u,v)|^2 \, du \, dv}{\iint_{\text{SLM}} |t_0(x,y)|^2 \, dx \, dy} \quad \text{or} \quad \eta_{E,\text{rel}} = \frac{\iint_{\text{SW}} |S(u,v)|^2 \, du \, dv}{\iint_{\text{SW} \cup \text{NW}} |S(u,v)|^2 \, du \, dv} \quad (4)$$

Note that both definitions assume a 100% fill factor, consequently representing the maximum case.

### 3. Hologram synthesis

SeeReal’s reconstruction scheme using viewing-window (VW) holography is illustrated in Fig. 1 [7]. When the SLM is illuminated by collimated coherent light, the 3D-scene is reconstructed in the frustum formed by the Fourier-transforming lens. The pixelated SLM generates a diffraction pattern in the Fourier plane. Its zero-order extension is the VW having an adapted size of $w_{x,y} = \lambda f/\delta_{x,y}$ slightly larger than the human eye pupil. Only within the VW the wave field seemingly emerging from the 3D object has to exist when the eye position is fixed or an active observer tracking is ensured. The eye pupil itself acts as spatial filter that isolates the encoded field from nonsignal or high-order terms and thereby enabling a high SNR.

Hologram synthesis means the computational calculation of the discrete complex wave-field $H(x,y)$ at the hologram plane that reconstructs the desired 3D-scene. The 3D-scene is represented by a sufficient number $n$ of points defined at discrete locations. Our implemented solutions for driving holographic displays with interactive or video content encoded in real-time are described in [8]. Basically, the computation methods for hologram synthesis can be grouped into direct analytic modeling by using ray-tracing and a Fourier-based modeling [9]. The first method treats each scene point $P_n(x_n, y_n, z_n)$ as a point emitter having a restricted angular spectrum according to the spatial resolution of the SLM and a defined brightness. The amplitude and phase distribution $H_n(x,y)$ of a single sub-hologram is then derived from the optical path between $P_n$ and the hologram plane. Finally, the complex amplitudes $H_n$ of all sub-holograms are summed up to the discrete hologram function $H(x,y) = \sum_{i=1}^{n} |H_n(x,y)| \exp [i \varphi_n(x,y)]$. In the second method the 3D-scene is sliced into layers ($L_1 \ldots L_m$) that run parallel to the SLM plane, where the size of each layer is limited by the frustum and the sampling count is the same as for the SLM. Object points $P_n$ are assigned to the closest layer and closest sampling point.
Each layer $m$ is then Fresnel-propagated to the Fourier plane and its complex distributions are summed up to form the signal $S(u, v)$ in the VW-plane. To obtain the hologram function $H(x, y)$ an inverse Fourier transform is applied to the signal, $H(x, y) = F^{-1}\{S(u, v)\}$.

4. Hologram representation on spatial light modulators

Numerous representation schemes have been developed since the beginning of computer-generated holography [10, 11]. The encoding step refers to the process of converting the complex amplitude of the desired reconstruction wave $U_R$ into a format, which can be displayed at the SLM by addressing its pixel. In synthetic holography, a fully complex representation is most qualified. The representation techniques common to synthetic holography can be differentiate into two categories:

Direct representation: The input wave $U_0$ is directly modulated by the hologram in such a way that the desired reconstruction wave $U_R$ emerges from the SLM. This corresponds to a multiplicative light manipulation, expressed by $U_R(x, y) = U_0(x, y)H(x, y)$. The hologram function $H(x, y)$ is then equal to the signal function $f(x, y)$, i.e. $H(x, y) = f(x, y)$. Most prominent example of this type is the kinoform.

Encoded representation: Here, the complex amplitude of the signal function $f(x, y)$ is not equal to the hologram function $H(x, y)$ but related via an addition function to $H(x, y) = F[f(x, y)]$ with $F$ being the encoding rule. The desired signal function must be physically decoded, which can be done by spatial filtering the signal term from noise terms in a Fourier plane of the hologram. Encoded representation methods have in common that the hologram is divided into discrete resolution cells (macro-pixel), which are further subdivided into sub-cells (sub-pixel). Usually, the signal function $f(x, y)$ is decomposed into pure phase or amplitude quantities that form the hologram function $H(x, y)$. However, this is compromised by a loss of spatial resolution. Examples of this type are detour-phase holograms [12–14] as well as double-phase holograms [15, 16].

The different encoding schemes discussed in this section are illustrated at the exemplary complex function $H(x, y)$ shown in Fig. 2, which has a sinusoidal amplitude and quadratic phase distribution.

$$H(x, y) = \sin(2\theta) \exp\left[i \left(10 \, (x^2 + y^2)\right)\right], \text{ with } -1 \leq (x, y) \leq 1 \text{ and } \theta = \arctan(y/x).$$

4.1. Burckhardt encoding

According to the method proposed by Burckhardt [14], a complex-valued function can be decomposed into three real and positive components. One hologram cell (macro-pixel) is laterally divided into three amplitude-modulating sub-cells, where the lateral shift between the sub-cells represents phase angles of 0°, 120° and 240° and acts as a phase offset. Thus,
Burckhardt holograms are detour-phase holograms that can be realized with an amplitude-modulating SLM. In holograms of this type, a phasor is decomposed into three vectors that run parallel to $\exp(i0) = 1$, $\exp(i2\pi/3) = -0.5 + i\sqrt{3}/2$ and $\exp(i4\pi/3) = -0.5 - i\sqrt{3}/2$, see Figure 3. Since the phase values are already represented by the lateral displacement of the sub-cells, any complex amplitude transmittance $H = |H| \exp(i\Phi)$ can be encoded in one macro-pixel as

$$H(x, y) = A_1(x, y) \exp[0] + A_2(x, y) \exp\left[\frac{2\pi}{3}\right] + A_3(x, y) \exp\left[\frac{4\pi}{3}\right], \quad (5)$$

where the magnitude $A_i$ of one term is always zero.

Figure 3. Burckhardt’s decomposition method into three real and positive components.

Advantage of this encoding scheme is that the decomposition can be done in an analytic way, which reduces the calculation cost. On the other hand, the diffraction efficiency of Burckhardt holograms is quite low. Notwithstanding, a fully complex modulation is achieved in a simple and easy way, i.e. by an amplitude-modulating SLM that comprises of an array of macro-pixels, albeit at the cost of reduced sampling and diffraction efficiency. Figure 4 shows reconstructions of 1D Burckhardt-encoded holograms displayed at a 20-inch direct-view color prototype from SeeReal.

4.2. Iterative phase-only encoding

Another encoded hologram representation employs the iterative Fourier transform algorithm (IFTA) [17–19], which can be adapted to not only create an intensity distribution but also
Fourier transform
\[ \text{FT}\{H(x,y)\} = U(u,v) \]

Inverse Fourier transform
\[ \text{FT}^{-1}\{U'(u,v)\} = H'(x,y) \]

Set amplitude constraints in hologram plane

Double-phase decomposition of \( H(x,y) \)

Calculated field
\[ H'(x,y) = |H(x,y)| \exp\left[i (x,y) \right] \]

Corrected field
\[ U'(u,v) = S \]

Corrected field
\[ H(x,y) = \exp\left[i \Phi \right] \]

End

Evaluation good enough?

Set spectrum constraints in Fourier plane

Corrected field
\[ U(u,v) = |U(u,v)| \exp\left[i \Phi(u,v) \right] \]

End

Evaluation good enough?

Figure 5. Flowchart of the adapted iterative Fourier transform algorithm.

Figure 6. Holographic reconstruction from an iterative phase encoded hologram displayed at a laboratory prototype. In the left image the camera focus is set to +200 mm (in front of display), in the right image the focus is at -1000 mm (behind display).

A desired complex distribution in the Fourier plane of the hologram. For the hologram representation problem discussed here, the targeting complex distribution \( S(u,v) \) in the Fourier domain is known, whereas a pure phase distribution in the hologram domain \( H(x,y) = \exp(i\Phi) \) that can be represented by a phase-only modulating SLM is searched for. The objective of the iterative algorithm is to find a Fourier transform pair that satisfies both the constraints in the hologram as well as the Fourier plane. A flowchart of the basic design algorithm is illustrated in Figure 5.

Starting point is the nominal complex amplitude of the signal function \( f(x,y) \), which is decomposed into two phase quantities by applying the double-phase hologram (DPH) approach [15, 16]. The Fourier transform of \( f(x,y) \) is the nominal complex distribution \( S(u,v) \) that serve as targeting function during optimization. The decomposition of \( f(x,y) \) in two phase-modulating pixel doubles also the matrix size in the Fourier plane when the Fourier transform is applied. But only half of the Fourier plane carries the desired signal to be optimized, which is therefore called the signal area. The iterative algorithm works as follows: (1) Apply the Fourier transform to the initial estimate of the hologram which yields the field \( U(u,v) \) in the Fourier plane; (2) Replace the resulting complex distribution within the signal area with the nominal complex distribution \( S(u,v) \) (set spectrum constraints); (3) Apply the inverse Fourier transform to the modified field \( U'(u,v) \) which gives a new field \( H'(x,y) \) in the hologram plane; (4) Replace the
modulus of the new calculated hologram $H'(x, y)$ with unity to provide a new hologram estimate (set hologram constraints). The iteration loop is repeated until the computed distribution in the Fourier domain satisfies the Fourier-domain constraints. Fidelity is evaluated by comparing the actual field in the signal area with the nominal signal field by use of the SNR-criterion as defined in Eq. (3).

Phase encoding has the benefits of higher efficiency and simple, single-layer SLM architecture. On the other hand, the iterative algorithm increases the computational costs and half of the native spatial resolution of the SLM is lost as one macro-pixel comprises of two sub-pixel. In Fig. 6 we demonstrate the huge depth range that can be realized by holography only. Full-parallax holograms with monochrome illumination are displayed on a laboratory prototype.

4.3. Two-phase interference encoding

By the double-phase hologram (DPH) approach, any complex number can be analytically decomposed as the sum of two pure phase values with constant magnitude as [15, 16]

$$H = |H| \exp(i\Phi) = \frac{1}{2} \exp[i(\Phi + \psi)] + \frac{1}{2} \exp[i(\Phi - \psi)]$$

(6)

where $\psi = \arccos |H|$ and $0 \leq \psi \leq \frac{\pi}{2}$. This holds for any arbitrary scalar complex field $H(x, y)$. Figure 7 shows the decomposition of $H$ in the complex plane. For example, two identical phase values with $\phi_1 = \phi_2$ give a resulting vector with maximum amplitude and phase of $\Phi$, whereas two phases with a phase difference of $\pi$ result in a vector with zero amplitude with non-defined phase. Arbitrary complex values are generated by combining other phases than those of the special cases $\Delta \phi = 0$ or $\pi$.

![Figure 7. Geometric representation of the double-phase decomposition.](image)

Only recently we introduced an implementation of a fully complex-modulating SLM that utilizes double-phase hologram (DPH) representation combined with two-beam interference [20]. To achieve a decoupled amplitude and phase modulation, two adjacent pixel of a phase-only modulating SLM are superimposed for interference by using polarization-sensitive components. The phase values to be addressed in neighboring pixel of the phase-SLM are $\phi_{1,2}(x, y) = \text{mod}\{\Phi(x, y) \pm \arccos |H|(x, y)|, 2\pi\}$.

Figure 8 shows the optical architecture of the complex SLM. Collimated TE-polarized light is spatially modulated in phase by an SLM and then passes through a structured half-wave plate (sHWP, isotropic/anisotropic zones), where the electric field vector of every second pixel column is rotated by 90° to TM-polarization, so that the sub-pixel to be composed to one macro-pixel have orthogonal polarization. The light fields of neighboring sub-pixel are brought to collinearity by a polarization-sensitive component (PSC), for example by a birefringent plate. The lateral displacement of the light field of every second pixel line must match the pixel pitch of the phase-modulation SLM in the selected combining direction. To bring them to interference, a subsequent linear polarizer with its transmission axis at 45° generates components of both polarizations from the two orthogonally polarized wave fields.
Figure 8. Optical architecture of the complex-modulating transmissive SLM sandwich.

Assuming completely polarized light and idealized components (no absorption or reflection), the Jones matrices for the two different paths through the sandwich (pixel 1, pixel 2) can be expressed as

\[ J_{P1} = J_{L+45P} \cdot J_{PSC} \cdot J_{sHWP1} \cdot J_{LC1} = \frac{1}{2} \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\phi_e} & 0 \\ 0 & e^{-i\phi_o} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\phi_1(V)} \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \]

\[ J_{P2} = J_{L+45P} \cdot J_{PSC} \cdot J_{sHWP2} \cdot J_{LC2} = \frac{1}{2} \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\phi_e} & 0 \\ 0 & e^{-i\phi_o} \end{pmatrix} \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ e^{-i\phi_2(V)} \end{pmatrix} \]

where \( \phi_o, \phi_e \) are the phase delays for the o- and e-wave of the birefringent plate and \( \phi_1, \phi_2 \) denote the voltage-dependent phase shifts of the two phase-modulating LC-SLM pixel. The resulting intensity after coherent beam superposition is then

\[ I(\phi_1, \phi_2) = E_\text{in}^\dagger (J_{P1} + J_{P2})^\dagger (J_{P1} + J_{P2}) E_\text{in} \]

with \( E_\text{in} \) the Jones vector for linear vertically polarized light.

Besides cost reasons, advantages of two-phase interference encoding scheme are the direct optical transform of pure phase values into complex values and a high diffraction efficiency. Complex modulation with a single active panel and otherwise passive optical components is feasible, which is however, compromised by a loss of half the spatial resolution in the direction of pixel combining. The hardware implementation enables a direct encoding of complex-valued hologram data without need of time-consuming iterations.

Figure 9. Holographic reconstructions taken with a digital SLR camera with focus (a) at foreground, \( z = +100 \) mm (letter ‘F’) and (b) at background, \( z = -100 \) mm (letter ‘B’).

Figure 9 presents holographic reconstructions from a laboratory prototype that uses a purpose-built complex-modulating SLM. The holographically generated 3D scene employs a single-parallax encoding in \( y \)-direction. Both phase and amplitude modulation is qualitatively demonstrated, which manifests in the clear depth discrimination (phase fidelity) and gray level representation (amplitude matching) of the reconstructions.
4.4. Comparison

When comparing the discussed encoding schemes, not only optical performance criteria but also SLM hardware architecture and calculation cost have to be considered. Most certainly, the intended application will determine the best choice of SLM and encoding. In some optoelectronic systems, a precomputing or delayed updating of the hologram might be acceptable. For real-time 3D imaging however, real-time capability is a must. Therefore, pixel-oriented encoding schemes that employ simple operations have a big advantage over global optimization methods. Table 1 summarizes key parameters of the encoding schemes discussed in this paper.

Table 1. Comparison of different encoding schemes.

<table>
<thead>
<tr>
<th>Modulation type</th>
<th>Number $M$ of sub-pixel</th>
<th>SBP</th>
<th>$\eta_{E,\text{abs}}$ maximum</th>
<th>$\eta_{E,\text{abs}}$ typical</th>
<th>Encoding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Burckhardt Amplitude</td>
<td>$M = 3$</td>
<td>$\frac{1}{3} (N_x N_y)$</td>
<td>$\leq 1.36%$</td>
<td>0.67%</td>
<td>direct</td>
</tr>
<tr>
<td>Iterative phase</td>
<td>$M \geq 2$</td>
<td>$\frac{1}{M} (N_x N_y)$</td>
<td>$\leq 50%$</td>
<td>50%</td>
<td>iterative</td>
</tr>
<tr>
<td>Two-phase interference</td>
<td>$M = 2$</td>
<td>$\frac{1}{2} (N_x N_y)$</td>
<td>$\leq 100%$</td>
<td>8.5%</td>
<td>direct</td>
</tr>
</tbody>
</table>

We have analyzed the three encoding schemes with regard to their sensitivity to quantization and addressing errors. Simulation was done with the exemplary complex function shown in Fig. 2, which was sampled with $640 \times 480$ points. In Fig. 10 (left) is shown how the SNR increases with the number of bit levels that were employed for discretization of the hologram function. Two-phase interference encoding exhibits the steepest increase, whereas the pure phase or amplitude representations approach their non-discretized maximum values with increasing bit level. Note that the SNR is also dependent from the total number of sampling points. Fig.10 (right) shows the SNR degradation in dependence from a percentage error up to 5\% (amplitude error for Burckhardt and phase error for the others) which was added to a 8-bit discretized hologram function. The Burckhardt encoding scheme is least sensitive to possible addressing errors. The tolerable error level depends on the targeting SNR.

![Signal-to-noise ratio](image1)

![Signal-to-noise ratio](image2)

**Figure 10.** Sensitivity of SNR to bit depth discretization (left) and to percentage error.

5. Summary

For dynamic applications such as three-dimensional holographic display, a full-complex modulation is highly desirable, because any restricted coding domain or operating curve results in reduced efficiency, noise terms, and complementary diffraction orders. The essentials of three different encoding schemes have been discussed and their implementation on prototypes using SeeReal’s proprietary Viewing-Window (VW) holography has been experimentally proven.
References


